Problema 154, Propuesto por José Luis Diaz Barrero, Barcelona, España

Haller todas las ternas $(x; y; z)$ de números reales que son solución del sistema de ecuacions

$$\left.\begin{array}{c} x+y+z=2 \\ \sqrt{2x+yz}+\sqrt{2y+zx}+\sqrt{2z+xy}=4 \end{array}\right\}$$

Solution by Kee-Wai Lau, Hong Kong, China

By the Cauchy- Schwarz inequality, we have

$$4=(x+y+z)^2 \le 3(x^2+y^2+z^2)$$

and

$$\begin{aligned} 16 = & \left(\sqrt{2x+yz}+\sqrt{2y+zx}+\sqrt{2z+xy}\right)^2 \\ \le & \ 3\left((2x+yz)+(2y+zx)+(2z+xy)\right) \\ = & \ 6(x+y+z)+\frac{3}{2}\left((x+y+z)^2-(x^2+y^2+z^2)\right) \\ = & \ 18-\frac{3}{2}\left(x^2+y^2+z^2\right) \\ \le & \ 18-\left(\frac{3}{2}\right)\left(\frac{4}{3}\right) \\ = & \ 16. \end{aligned}$$

It follows that $(x; y; z) = (\frac{2}{3}; \frac{2}{3}; \frac{2}{3})$.

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