

Problema 178.

Sean a, b, c números reales verificando $0 < a, b, c \leq 1$. Demostrar que

$$\frac{1}{\sqrt{2+3a}} + \frac{2}{\sqrt{2+3b}} + \frac{3}{\sqrt{2+3c}} \geq \frac{6\sqrt{5}}{5}.$$

Claramente

$$\frac{1}{\sqrt{2+3a}} \geq \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{2+3b}} \geq \frac{2}{\sqrt{5}}, \frac{3}{\sqrt{2+3c}} \geq \frac{3}{\sqrt{5}},$$

sumando

$$\frac{1}{\sqrt{2+3a}} + \frac{2}{\sqrt{2+3b}} + \frac{3}{\sqrt{2+3c}} \geq \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$$